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Letter

On two distinct Reynolds number regimes of a turbulent square jet

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ABSTRACT

The effects of Reynolds number on both large-scale and small-scale turbulence properties are investigated in a square jet issuing from a square pipe. The detailed velocity fields were measured at five different exit Reynolds numbers of $8 \times 10^3 \leq Re \leq 5 \times 10^4$. It is found that both large-scale properties (e.g., rates of mean velocity decay and spread) and small-scale properties (e.g., the dimensionless dissipation rate constant $A = \varepsilon L/\langle u^2\rangle^{3/2}$) are dependent on Re for $Re \leq 3 \times 10^4$ or $Re_\lambda \leq 190$, but virtually become Re-independent with increasing Re or Re_λ. In addition, for $Re_\lambda > 190$, the value of $A = \varepsilon L/\langle u^2\rangle^{3/2}$ in the present square jet converges to 0.5, which is consistent with the observation in direct numerical simulations of box turbulence, but lower than that in circular jet, plate wake flows, and grid turbulence. The discrepancies in critical Reynolds number and $A = \varepsilon L/\langle u^2\rangle^{3/2}$ among different turbulent flows most likely result from the flow type and initial conditions.

Introduction Turbulent jets are applied in various industrial mixing processes, e.g., dispersal and combustion. In these flows, large-scale motions whose scale is close to global flow scale contain most of the kinetic energy and dominate the transfer of momentum, heat and mass, while the small-scale turbulence spanning the dissipative and inertial ranges brings different species together at the molecular level [1,2].

Previous investigations by Dimotakis [3,4] indicated that a mixing transition, beyond which the amount of mixed fluid become Re-independent for $Re > Re_{cr}$ and the flow becomes fully developed turbulence, occurs in jets and other shear flows. This can be observed widely in turbulence. Here Re_{cr} is a critical Reynolds number. Dimotakis claimed that, the fully-developed turbulence, the existence of a range of scales (uncoupled from the large scales and free from the viscosity effect) is a necessary condition. A outer-scale Reynolds number $Re = U\delta/\nu > 10000$–20000 or a Taylor Reynolds number $Re_\lambda = \langle u^2\rangle^{1/2}\lambda/\nu \geq 100$–$140$ is required by the resulting fully-developed turbulent. Here u represents the longitudinal component of the fluctuating velocity, $\lambda \equiv \langle u^2\rangle^{1/2}\langle(\partial u/\partial x)^2\rangle^{-1/2}$ and ν the kinematic viscosity. This observation is supported by Fellouah and Pollard [5] and Mi et al. [6] in their investigations of circular jets. In addition, Mi et al. [6] suggested that ε may be well estimated by $\varepsilon = A_1\langle u^2\rangle^{3/2}/L$ for $Re \geq 10000$ or $Re_\lambda \geq 110$ and $\varepsilon = A_2\nu\langle u^2\rangle/L^2$ for $Re < 10000$ or $Re_\lambda < 110$; here L is the integral length-scale of turbulence while A_1 and A_2 are experimental constants.

Compared to circular jets, noncircular jets have been found more effective in mixing with ambient fluid [7,8]. In the case of square jets, Xu et al. [9] measured square jet flows emanating from a long square tube using hot wire measurements in the range of $8000 \leq Re \leq 50000$. They found that all the far-field rates of the mean velocity decay and spread, and the asymptotic value of the streamwise turbulent intensity, decrease as Re increases for $Re \leq 3 \times 10^4$, while they become almost Re-independent for $Re > 3 \times 10^4$. However, Xu et al. [9] did not provide information on the influence of Reynolds number on small-scale turbulent properties of square jets. In this sense, is there any difference between the critical transition Reynolds numbers for large-scale and small-scale turbulences in the square jet? In addition, the critical Reynolds number in the square jet seems higher than that of circular jets. What is(are) the reason(s) for the critical Reynolds number varying from flow to flow? To address these important questions, we conduct the present study to investigate the effects of Reynolds number on both large-scale and small-scale turbulent properties of a square jet at five different Reynolds numbers between 8000 and 50000.

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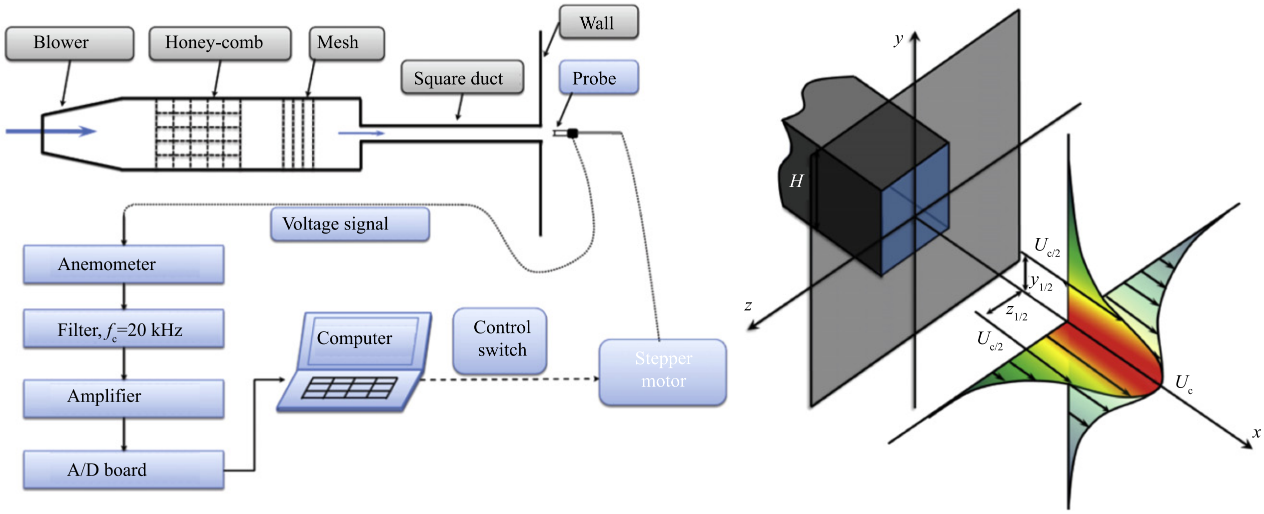

Fig. 1. (a) Experimental setup and (b) 3D square jet nozzle exit, notations, and coordinate system. $U_{c/2}$ is the axial mean velocity at either $Y_{1/2}$ or $Z_{1/2}$.

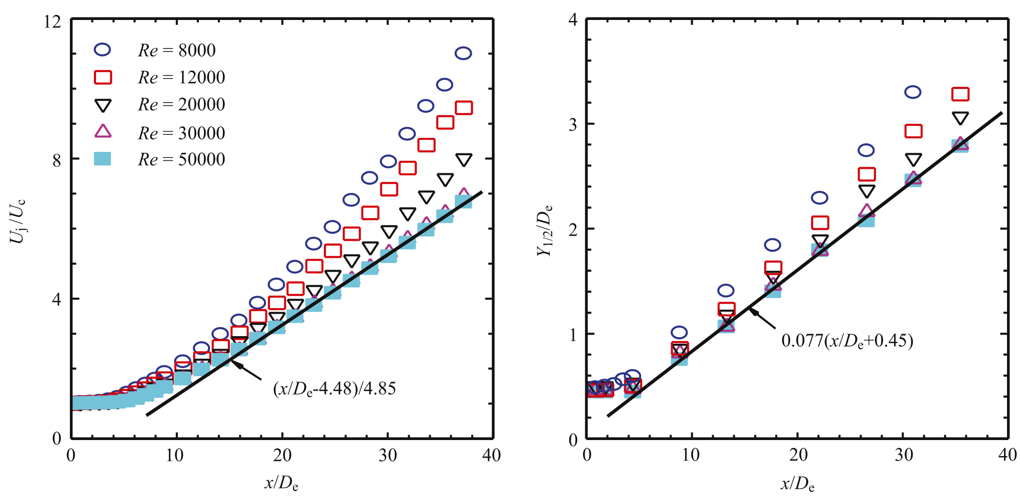

Fig. 2. Streamwise variations of (a) inverse centerline mean velocity U_j/U_c and (b) normalized half-width in the x–y plane $Y_{1/2}/D_e$ for $Re = 8000$–50000.

Content Detailed description of the present measurement is referred to Xu et al. [9], only a brief version is provided here and the data processing methods for small-scale turbulent statistics are carefully introduced in Mi et al. [10]. The present square jets were generated from a nozzle system whose schematic diagram is shown in Fig. 1(a). The present facility consisted of a square duct with the width of the square duct $H = 2.5$ cm, and the length of approximately 2 m. The nominal opening area $A = 6.25$ cm^2 and the equivalent diameter $D_e \equiv 2(A/\pi)^{1/2}$ was approximately 2.82 cm. The mean streamwise velocity U_j at the center of the square exit plane was varied over the range $4.2 \le U_j \le 26.4$ m/s, corresponding to a Reynolds number range $8 \times 10^3 < Re < 5 \times 10^4$, with $Re \equiv U_j D_e/\nu$. With $f_c = 20$ kHz (an identical cut-off frequency), velocity signals were low-pass filtered for all measurements to eliminate excessively high-frequency noise and to avoid aliasing. The voltage signals were then digitized on a personal computer at $f_s = 40$ kHz via a 12 bit A/D converter and each record had a duration of about 80 s. The present study corrected the spatial attenuation of the single wire due to the wire length $\lambda_w \approx 1$ mm using the procedure of Wyngaard [11], which was developed in spectral space to account for the integration effect on Fourier components of the velocity. To remove the effect of high frequency noise, the present data were filtered using the digital scheme of filtering high-frequency noise proposed by Mi et al. [10,12]. In this context, the present study estimates ε from hot-wire measurements of $u(t)$, using the isotropic relation $\varepsilon = 15\nu\langle(\partial u/\partial x)^2\rangle$ together with modified Taylor's hypothesis $\partial u/\partial x = (U_c + u)^{-1}\partial u/\partial t$, rather than $U_c^{-1}\partial u/\partial t$ [13].

Figure 2 presents the streamwise variation of the inverse centerline mean velocity U_c normalized by the exit centerline mean velocity U_j, i.e., U_j/U_c and the normalized half width $Y_{1/2}/D_e$ in the range of $8000 \le Re \le 50000$. To quantitatively study the dependence of U_j/U_c and $Y_{1/2}/D_e$ on Re, the well-known self-preserving relations are applied, i.e.

$$U_j/U_c = [(x - x_U)/D_e]/K_U, \quad (1)$$

$$Y_{1/2}/D_e = K_Y[(x - x_Y)/D_e], \quad (2)$$

where K_U and K_Y are the jet velocity decay rate and spread rate, x is axial downstream distance measured from the nozzle exit, and x_U and x_Y are the x-locations of the virtual origin of Eqs. (1) and (2). Figure 3 shows the variations of the jet velocity decay rate K_U and spread rate K_Y with the Reynolds number. K_U and K_Y significantly depend on Re for $Re \le 3 \times 10^4$, as demonstrated clearly in Fig. 3, but appear to independent with further increasing Re. Thus, there seems to be a critical Reynolds number of $Re_{cr} = 30000$ for the large-scale turbulence in the present square jet flow.

To investigate the influence of Re on small-scale turbulence of the present square jet, Fig. 4 presents the streamwise evolution of the normalized dissipation rate $\varepsilon^* = \varepsilon D_e/U_j^3$ in the range of

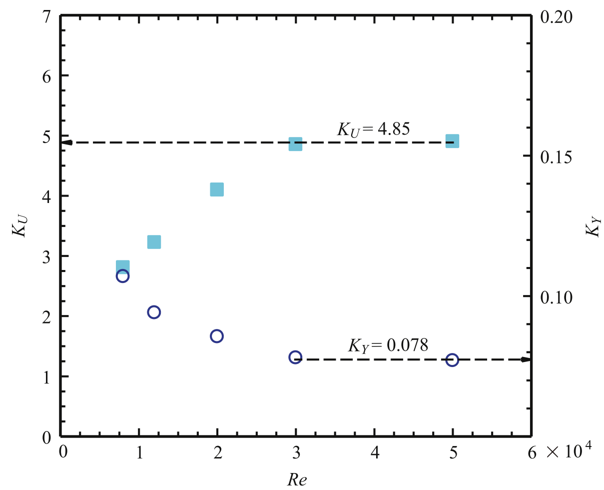

Fig. 3. *Re*-dependence of the mean velocity decay K_U and spread rates K_Y.

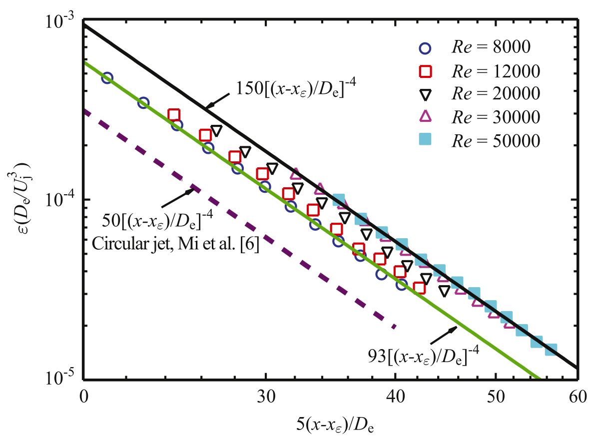

Fig. 4. Streamwise evolution of the normalized energy dissipation rate $\varepsilon^* = \varepsilon D_e/U_j^3$. The data from Mi et al. [6] is also added for comparison.

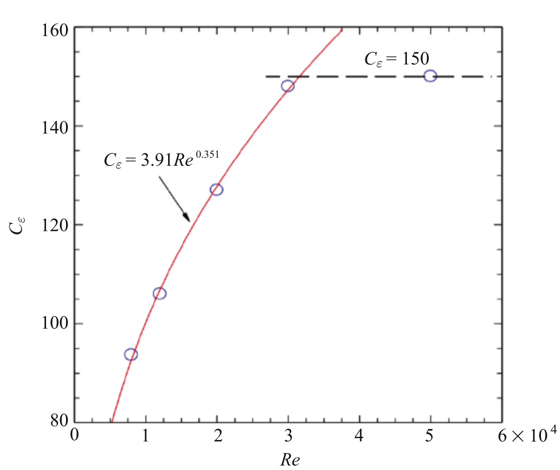

Fig. 5. Dependence on *Re* of the prefactor C_ε.

$Re = 8000$–50000. Clearly, as the jet flow develops downstream, ε^* decreases rapidly with downstream distance x and follows the self-preserving relation of the mean energy dissipation rate in the self-preserving circular jet [6], i.e.

$$\varepsilon^* = \varepsilon(D_e/U_j^3) = C_\varepsilon[(x - x_\varepsilon)/D_e]^{-4}, \qquad (3)$$

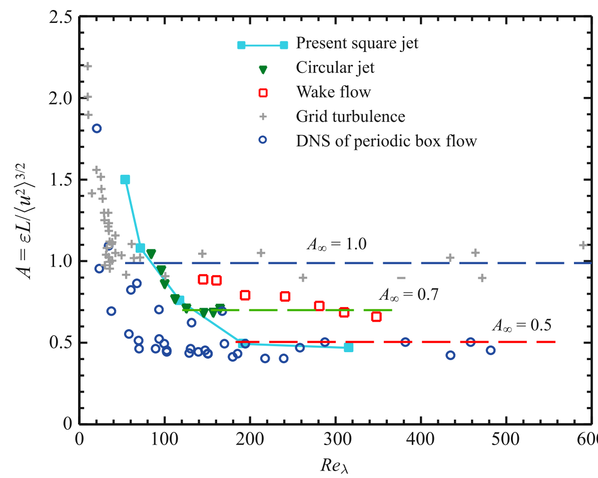

Fig. 6. Dependence of $A = \varepsilon L/\langle u^2\rangle^{3/2}$ on Re_λ. Symbols: ■–■, present square jet; ▼, circular jet turbulence [6]; +, grid turbulence, compiled by Sreenivasan [14]; ○, direct numerical simulation (DNS) of periodic box turbulence, compiled by Burattini et al. [15]; □, wake flow [15].

where C_ε is the prefactor, and x_ε is the virtual origin location. The value of C_ε can be determined by fitting measured data with Eq. (3), which is shown in Fig. 5. For $Re \le 3 \times 10^4$, as observed from Fig. 6, the prefactor (C_ε) of Eq. (3) increases with *Re*. By fitting the value of C_ε for $Re \le 3 \times 10^4$, the relationship between C_ε and *Re* is $C_\varepsilon \approx 3.91Re^{0.351}$. For $Re > 3 \times 10^4$, all the measured data of ε^* becomes constant and collapses virtually onto a single horizontal line with $C_\varepsilon \approx 150$, suggesting that ε^* becomes nearly independent of the Reynolds number. Thus, the critical transition Reynolds numbers for both large-scale and small-scale turbulence of the square jet is the same, i.e., $Re_{cr} \approx 30000$. However, this value is higher than that in a circular jet, whose critical Reynolds number Re_{cr} is about 10000, across which the jet turbulence behaves distinctly, see Ref. [6]. In addition, it is worth noting that the normalized energy dissipation rate ε^* and the prefactor C_ε are higher for the present square jet than the circular jet measured in Mi et al. [6]. It indicates that the mean turbulent energy is dissipated at higher rate and mixing is enhanced for the square jet, compared to the circular jet.

Further, the results $A = \varepsilon L/\langle u^2\rangle^{3/2}$ estimated from the present square jet flow for $Re = 8000$–50000 or $Re_\lambda = 54$–316 are shown in Fig. 6. A number of previous datasets for circular jet, grid turbulence, wake flows, DNS of box turbulence are also added for comparison. It is interesting to note that the value of A deceases from 1.5 to 0.5, as Re_λ increases from 54 to 190. For $Re_\lambda \ge 190$, the value of A seems to asymptote to a constant value of 0.5, denoted by A_∞. In a circular jet, Mi et al. [6] found that, for $Re_\lambda < 130$, A obviously decreases notably with increasing Re_λ, while A becomes nearly independent of Re_λ at $Re_\lambda \ge 130$. Sreenivasan [14] checked the dependence of A on Re_λ over a greater range of Re_λ through collecting a number of previous datasets for grid turbulence produced by biplane square meshes, and found that the critical Reynolds number $Re_{\lambda,cr} \approx 50$ for grid turbulence. For DNS of period box flow, $Re_{\lambda,cr}$ seems to be about 200, see Fig. 6. The present results show that the critical Reynolds number is unlikely to lie in just a narrow range of Reynolds numbers as suggested by Dimotakis [3] generally for any turbulent flows. Figure 6 also demonstrates that A_∞ differs appreciably for various flows. Explicitly indicated on the plot are $A_\infty \approx 0.5$ for the present square jet and DNS of periodic box turbulence [15]; $A_\infty \approx 0.7$ for a circular jet [6] and plate wake [15]; $A_\infty \approx 1.0$ for the grid turbulence [14,16].

The above discrepancies in A_∞ and critical Reynolds number are most likely to result from the flow type and initial conditions [6]. This indicates that $\langle u^2\rangle^{3/2}/L$ is not proportional to the rate at which energy transferred from the large-scale eddies [17]. Mazellier and Vassilicos [18] indicated that the nonuniversal asymptotic of the dimensionless dissipation rate constant A stems from its universal dependence on the number of large-scale eddies, which strongly varies from flow to flow. Compared to the smooth contraction circular jet studied by Mi et al. [6] and wake flow in Ref. [15], the long pipe square jet produces a power-law profile of the mean velocity and a very thick fully-turbulent boundary-layer at exit, thus resulting in the less large-scale eddies in the near field and smaller A_∞. Despite A_∞ and $Re_{\lambda,\mathrm{cr}}$ varying for different flows, according to Fig. 6, $A = \varepsilon L/\langle u^2\rangle^{3/2}$ in general decreases with increasing Re_λ until $Re_\lambda = Re_{\lambda,\mathrm{cr}}$. For $Re_\lambda > Re_{\lambda,\mathrm{cr}}$, there is a good constancy of $\varepsilon L/\langle u^2\rangle^{3/2}$, i.e., $A = A_\infty$ with almost constant of order unity.

This study has successfully clarified by experiments the effects of Reynolds number on both the large-scale and small-scale turbulence properties from the transition region to the self-preserving far field of a square jet. Consistent with the large-scale properties (e.g., the centerline mean velocity and half-width), the small-scale properties (e.g., the normalized mean dissipation rate) of the square jet have been found to significantly depend on Reynolds number for $Re \leq 30000$ or $Re_\lambda < 190$, but weaken with further increasing Re or Re_λ. For $Re_\lambda > 190$, the value of $A = \varepsilon L/\langle u^2\rangle^{3/2}$ in the present square jet converges to 0.5, consistent with the observation in DNS of box turbulence, but lower than that in circular jet, grid turbulence and wake flows. The discrepancies in $A = \varepsilon L/\langle u^2\rangle^{3/2}$ and critical Reynolds number among different turbulent flows are most likely to result from the flow type and initial conditions. Compared to the smooth contraction circular jet and wake flow, the long pipe square jet produces less large-scale eddies, thus resulting in smaller value of the asymptotic of the dimensionless dissipation rate constant.

Acknowledgments

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