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Observer-based Event-triggered Circle Formation Control for Multi-agent Systems with Directed Topologies

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***Abstract*—This paper proposes an observer-based event-triggered algorithm for circle formation control problems of first-order multi-agent systems, where the communication topology is modeled by a spanning tree-based directed graph with limited resources. Depending on the trigger threshold of specific measurement error and compared with the norm of a function with states, we apply an event-triggered mechanism to reduce the updates frequency of the controller via observing continually neighbors' state. Sufficient conditions on desired circle formation are derived following the resulting asynchronous network executions converge to the equilibrium points. Both the analysis and numerical simulations show that Zeno behavior can be ruled out under the designed control laws.**

***Keywords*—Multi-agent Systems; Circle Formation; Event-triggered; Directed Network**

I. Introduction

In recent years, many research works have been done on controlling of multi-agent systems (MASs) due to both its practical potentials [1], [2] and theoretical challenges [3], [4], [5]. As a significant issue in cooperative control for MASs, formation control, aiming at guiding multiple agents to form and maintain predetermined geometries, has attracted considerable interests for its extensive applications in different fields [6], [7], [8]. The primary attention has been devoted to the design of distributed formation control framework, especially concerning the increasing number of agents and the robustness against internal uncertainties and external disturbances[9], [10]. Therefore, most current research results on formation control mainly rely on the following ideal hypothesis [11], [12], [13]. For example, each agent is modeled as unlimited communication capabilities, unlimited power, and unlimited processing capabilities, which allows arbitrary information exchange patterns. However, in really systems, robots usually have power constraints and limited capacity of communication.

In order to save energy and bandwidth for pratical applications, event-triggered control methodology has been proposed [14], [15], [16], [17], [18]. The most distinct character of event-triggered control is that control actions are updated only when specific events occur and the trade-offs among actuator effort, communication, and computation are eased. A simple state event-triggered schedule based on the feedback controller was studied in [14], which leads to a guaranteed performance with a fixed sampling rate requirements concerning the optimizing schedules and sampling rates. In [15], under conditions of exponentially decreasing thresholds on the measurement errors, a time-dependent triggering method was designed to guarantee asymptotic convergence to a ball centered at the average consensus.A distributed model-based approach was derived under a class of the networks of nonlinear dynamical agents to ensure the synchronization of the overall system[16]. Furthermore, [19] combined with event-triggered protocols to solve circle formation problems for first-order MASs. Also, [20], [21] investigated a combination algorithm based on quantized communication technology, where the problem of MASs with a limitation of communication was addressed. Given the above reviews, it is noteworthy tha most of the

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existing results on event-triggered control are meant to prevent the case of Zeno behaviors [22], [23], such that a finite number of samplings generate within a finite amount of time. Regularly, a sufficient condition to exclude Zeno behavior is to ensure the event trigger interval is strictly positive lower bounded [24].

Different from previous studies, especially in [19], [20], the main objective of this paper is to provide a novel control method to solve circle formation problem for first-order MASs through a set of directed graphs. In this studies, similar to Pioneer 3-DX in [25], each agent perceives distance through communication from counterclockwise to its nearest neighbor, and the counterpart in clockwise. The main contributions of this paper are listed as below. Firstly, combining with a distributed asynchronous event-triggered control methodology, a novel control method is designed to solve the circle formation problem of first-order dynamics MASs. Secondly, based on the analysis in [26], the proposed strategy allows for a reduction of the number of control actions without significantly degrading performance. At last, the resulting asynchronous model achieves an asymptotically desired equilibrium point while the absence of Zeno behavior is guaranteed, i.e., no trajectory is generated in a finite time interval.

The remainder of this paper is organized as follows. Preliminary definitions and the problem formulation are presented in Section II. In Section III, a distributed circle formation control law for first-order MASs and the rigorous analysis of its performance are given. Section IV discusses the simulation results before we conclude in Section V.

II. Preliminaries and Problem Statement

In this section, we first introduce some notions and concepts used througn out the paper, and then formulate a concerned circle formation problem.

A. Preliminaries

For a finite set $\mathcal{S}$, $|\mathcal{S}|$ denotes the number of its elements. For a vector or a matrix A, $\|A\|$ stands for its $\mathrm{Euclidean}$ norm, $\|A\|_\infty$ stands for its ∞-norm and A^T is its transpose. $\mathbf{1}_N$ and $\mathbf{0}_N$ are the N dimension column vectors with all entries 1 and 0, respectively. The matrix $\mathrm{diag}\{a_1, a_2, ..., a_N\}$ denotes the diagonal matrix with diagonal entries $a_1, a_2, ..., a_N$.

The notation $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ is a directed graph, where $\mathcal{V} = \{1, 2, ..., N\}$ is a set of nodes, $\mathcal{E} = \mathcal{V} \times \mathcal{V}$ denotes a set edges, and $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N\times N}$ stands for a weighted adjacency matrix, where $\mathbb{R}$ denotes real numbers. In $\mathcal{G}$, for all $i \in \mathcal{V}$, $(i, i) \notin \mathcal{E}$. Namely, edge $(j, i) \in \mathcal{E}$, starting from node j and ending to node i, indicates agent i can perceive state information from agent j. Furthermore, agent j is called an in-neighbor of agent i, and $\mathcal{N}_i = \{j \in \mathcal{V} \,|\, (j, i) \in \mathcal{E}\}$ is used to represent the in-neighbors set of agent i. Especially, the edge (i, j) links with the elements a_{ij} of a weighted adjacency matrix $\mathcal{A}$, $a_{ij} > 0$ if and only if $(i, j) \in \mathcal{E}$, otherwise $a_{ij} = 0$. we use $\mathbf{d}_i = \sum_{j=1}^N a_{ij}$ to denote the in-degree of agent i in $\mathcal{G}$, and then define $\mathcal{L} = \mathcal{D} - \mathcal{A}$ as Laplacian matrix of $\mathcal{G}$, where $\mathcal{D} = \mathrm{diag}\{\mathbf{d}_1, \mathbf{d}_2, ..., \mathbf{d}_N\}$. Subsequently, we list the eigenvalues of $\mathcal{L}$ in a descending order: $\lambda_N \geq ... \geq \lambda_2 \geq \lambda_1 = 0$, where λ_N denotes the spectral radius of $\mathcal{L}$.

The following lemmas are used to facilitate the analysis.

Lemma 1. *([27]) For any $x, y \in \mathbb{R}$ and $a > 0$, it has the following properties*

$$1. xy \leq \frac{a}{2}x^2 + \frac{1}{2a}y^2;$$
$$2. (x^2 + y^2) \leq (x + y)^2, if \quad xy \geq 0.$$

Lemma 2. *([28]) Given a directed graph $\mathcal{G}$, composed of a spanning tree, the vector $\xi = [\xi_1, \xi_2, ..., \xi_N]^T > 0$ satisfies $\sum_{i=1}^N \xi_i = 1$ and $\xi^T \mathcal{L} = \mathbf{0}_N$, in which ξ denotes the left eigenvector corresponding to zero eigenvalue of the* Laplacian *matrix $\mathcal{L}$. Furthermore, $\mathcal{L}^T\Theta + \Theta\mathcal{L}^T$ is semi-positive definite, where $\Theta = \mathrm{diag}\{\xi_1, \xi_2, ..., \xi_N\}$. Taking extractions of the square root of each element of Θ, a matrix is obtained as $\Upsilon = \mathrm{diag}\{\gamma_1, \gamma_2, ..., \gamma_N\}$, where $\gamma_i = \sqrt{\xi_i}, i = 1, ..., N$.*

B. Problem formulation

Consider a MAS with N ($N \geq 2$) mobile agents, as shown in Fig. 1, the agents are initially located on a predefined circle, and no two agents occupy the same position simultaneously. For simplicity, the agents are labeled counterclockwise, and the position of each agent $i \in \{1, 2, ..., N\}$ is measured by angles $x_i(t)$. In general, the initial positions of agents are set to follow the rule

$$0 \leq x_1(0) < ... < x_i(0) < x_{i+1}(0) < ... < x_N(0) < 2\pi. \tag{1}$$

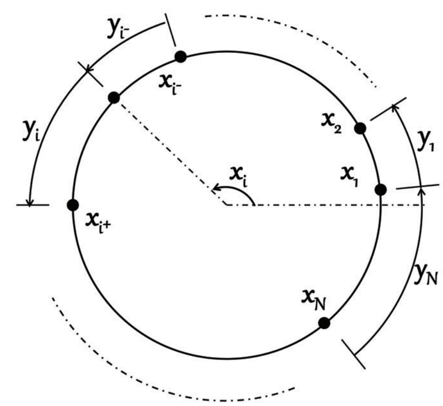

Fig. 1. Agents are distributed on a circle.

In this case, each agent only has two neighbors, $\mathcal{N}_i = \{i^+, i^-\}$ is given to denote two neighbors of mobile agent i, where

$$i^+ = \begin{cases} i + 1, & \text{when} \quad i = 1, 2, ..., N - 1, \\ 1, & \text{when} \quad i = N, \end{cases} \tag{2}$$

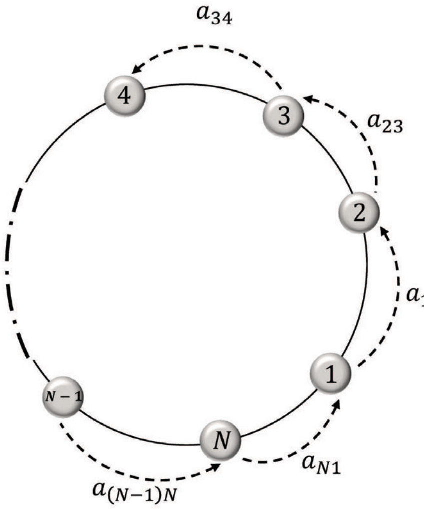

Fig. 2. A strongly connected digraph $\mathcal{G}$ with N agents.

and

$$i^- = \begin{cases} N, & \text{when} \quad i = 1, \\ i-1, & \text{when} \quad i = 2,3,...,N. \end{cases} \tag{3}$$

Then, the relationships of information exchange among agents further being established in such a topology, in which each agent i with a micro-sensor can only collect the the angular distance from i and i^+. On the other hand, the counterpart from i and i^- is obtained via a shared communication topology. In this case, the communication topology can be described as shown Fig. 2.

The dynamics of each agent for circle formation control is given as

$$\dot{x}_i(t) = u_i(t), \quad i \in \mathcal{V}, \tag{4}$$

where $x_i \in \mathbb{R}$ is the scalar state of agent i, and $u_i \in \mathbb{R}$ is the control input of agent i.

Based on counterclockwise at time t, $y_i(t) \in \mathbb{R}$ is given as the angular distance from agent i to agent i^+ measured by agent i. Along with rules (2) and (3), it yields

$$y_i(t) = \begin{cases} x_{i^+}(t) - x_i(t), & \text{when} \quad i = 1,2,...,N-1, \\ x_{i^+}(t) - x_i(t) + 2\pi, & \text{when} \quad i = N, \end{cases} \tag{5}$$

where the stack vector $y(t) = [y_1(t), y_2(t), ..., y_N(t)]^T \in \mathbb{R}^N$, and $\sum_{i=1}^N y_i(t) = 2\pi$ always holds.

Assume the desired circle formation is determined by vector

$$d = [d_1, d_2, ..., d_N]^T,$$

where $d_i \in \mathbb{R}$ stands for the desired angular distance between agent i and agent i^+. A desired circle formation is admissible if and only if d satisfies $d_i > 0$ and $\sum_{i=1}^N d_i = 2\pi$, $i = 1,2,...,N$.

The definition of the circle formation problem for first-order MASs is described as

Definition 1. *(Circle Formation Problem of First-order MASs) Given an admissible circle formation characterized by d, a distributed control law $u_i(t, y_i(t)), i = 1,2,...,N$ is designed, such that the solution to system (4) converges to some equilibrium point x^* under any initial condition (1). That is, $y^* = d$ satisfies.*

III. Observer-based event-triggered control law

According to the sampled-date based way-point control law designed in [7], given as

$$u_i(t) = \frac{d_{i^-}}{d_i + d_{i^-}} y_i(t) - \frac{d_i}{d_i + d_{i^-}} y_{i^-}(t), \quad t \geq 0, \tag{6}$$

From [7], we know that the continuous updating control law (6) can drive all agents move to their equilibrium point x^*, however typically wasting unnecessary transmission energy and communication bandwidth. In order to solve this problem, an observer-based event-triggered strategy is proposed. Note that the controllers of agents only update at discrete event instants, where continuous communication between neighboring agents maintains. Furthermore, intermediate variables are introduced as the increasing sequence $t_0^i, t_1^i, ..., t_k^i, ...$ to denote event instants of agent i, such that $y_i(t_k^i)$ is the state of agent i at the kth event instant. Therefore, each agent has its event sequence because all agents are triggered asynchronously.

According to the event-triggered strategy, the distributed circle formation control law for agent i is designed as

$$u_i(t) = \frac{d_{i^-}}{d_{i^-} + d_i} y_i(t_k^i) - \frac{d_i}{d_{i^-} + d_i} y_{i^-}(t_{k'}^{i^-}), t \in [t_k^i, t_{k+1}^i), \tag{7}$$

where $t_{k'}^{i^-} \triangleq \arg\min_{l \in \mathbb{N}, t \geq t_l^{i^-}} \{t - t_l^{i^-}\}$ represents the last event instant of agent i^-.

From (7), the controller of agent i updates at its own event sequence $(t_0^i, t_1^i, ..., t_k^i, ...)$. We define $\hat{y}_i(t) = y_i(t_k^i)$, $\hat{\delta}_i(t) = \frac{\hat{y}_i(t)}{d_i}$. Therefore, the control law (7) can be written as

$$u_i(t) = \frac{d_i d_{i^-}}{d_{i^-} + d_i} (\hat{\delta}_i(t) - \hat{\delta}_{i^-}(t)), t \in [t_k^i, t_{k+1}^i). \tag{8}$$

Replacing (5) and (8) into (4), the closed-loop of agent i can be rewritten by δ_i as

$$\dot{\delta}_i(t) = \sum_{j \in \mathcal{N}_i} \frac{d_j}{d_i + d_j} \left(\hat{\delta}_j(t) - \hat{\delta}_i(t) \right), t \geq 0. \tag{9}$$

Define a deviation variable $e_i(t) = \hat{\delta}_i(t) - \delta_i(t)$, then a compact form of the system dynamics can be written as

$$\dot{\delta}(t) = -L_d^T(\delta(t) + e(t)), t \in [t_k^i, t_{k+1}^i), \tag{10}$$

where $\delta(t) = [\delta_1(t), \delta_2(t), ..., \delta_N(t)] \in \mathbb{R}^N$, $e(t) = [e_1(t), e_2(t), ..., e_N(t)] \in \mathbb{R}^N$,

Based on the designed control law (8) and system (10), the event-triggered circle formation control for the distributed MAS is solved by the following theorem.

Theorem 1. *Given any admissible circle formation characterized by d, and take into consideration system (10) and*

$$L(d)=\begin{bmatrix} \frac{d_2}{d_2+d_1}+\frac{d_N}{d_N+d_1} & -\frac{d_1}{d_2+d_1} & 0 & \dots & 0 & -\frac{d_1}{d_N+d_1} \\ -\frac{d_2}{d_2+d_1} & \frac{d_3}{d_3+d_2}+\frac{d_1}{d_2+d_1} & -\frac{d_2}{d_3+d_2} & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & \frac{d_N}{d_N+d_{N-1}}+\frac{d_{N-2}}{d_{N-1}+d_{N-2}} & -\frac{d_{N-1}}{d_N+d_{N-1}} \\ -\frac{d_N}{d_N+d_1} & 0 & 0 & \dots & -\frac{d_N}{d_N+d_{N-1}} & \frac{d_1}{d_N+d_1}+\frac{d_{N-1}}{d_N+d_{N-1}} \end{bmatrix}. \tag{11}$$

the designed control law (8) *over a strongly connected weight-unbalanced digraph* $\mathcal{G}$*, the circle formation problem is solvable when the observer-based event-trigger condition is designed as*

$$f_i(t)=\|e_i(t)\|-\frac{\sigma\|\gamma_i\bar{\delta}_i\|}{\|\Upsilon L_d^T\|}, 0<\sigma<1, \tag{12}$$

where $\bar{\delta}_i$ *is the* i*th elements of* $\bar{\delta}=[\bar{\delta}_1,\bar{\delta}_2,...,\bar{\delta}_N]^T\triangleq L_d^T\delta$*,* Υ *is the same diagonal matrix as described in Lemma 2,* γ_i *is the* i*th diagonal element of matrix* Υ*. Moreover, there exists at least one agents* $m\in\mathcal{V}$ *in system (10), which prevents occurrence of* Zeno *behavior under event-trigger condition (12).*

Proof. To show the effectiveness of the proposed control law, Lyapunov function in [29] is considered

$$V(t)=\frac{1}{4}\delta^T(t)(L_d\Theta+\Theta L_d^T)\delta(t), \tag{13}$$

where Θ is the same diagonal matrix as Lemma 2, such that $L_d\Theta+\Theta L_d^T$ is semi-positive definite.

As a result, $V(t)\le 0$ and $V(t)=0$ if the circle formation problem is solvable. The derivative of the Lyapunov function (13) along of the trajectories yields

$$\begin{aligned}\dot{V}(t)&=\delta^T(t)L_d\Theta(-L_d^T(\delta(t)+e(t)))\\&=-\delta^T(t)L_d\Theta L_d^T\delta(t)-\delta^T(t)L_d\Theta L_d^Te(t)\\&\le-\|\Upsilon L_d^T\delta(t)\|^2+\|\Upsilon L_d^T\delta(t)\|\|\Upsilon L_d^Te(t)\|.\end{aligned} \tag{14}$$

Enforcing the event condition in (12), $\|\Upsilon L_d^Te(t)\|\le\|\Upsilon L_d^T\|\|e(t)\|\le\sigma\|\Upsilon L_d^T\delta\|$. Thus, (14) is rewritten as

$$\begin{aligned}\dot{V}(t)&\le\|\Upsilon L_d^T\delta(t)\|^2(\sigma-1)\\&\le\|\Upsilon\bar{\delta}(t)\|^2(\sigma-1).\end{aligned} \tag{15}$$

Obviously, $\bar{\delta}(t)$ can be calculated by observing continually neighbors' states. As $0<\sigma<1$, $\dot{V}(t)\le 0$ and $\dot{V}(t)=0$ if the circle formation problem is solvable.

According to the Lemma 2, we have

$$\sum_{i=1}^N\xi_i\delta_i(t+1)=\sum_{i=1}^N\xi_i\delta_i(t)=...=\sum_{i=1}^N\xi_i\delta_i(0).$$

Combining (7) and (12), all conditions lead to

$$\lim_{t\to\infty}\delta_i(t)=\lim_{t\to\infty}\delta_j(t)=\sum_{i=1}^N\xi_i\delta_i(0)=c, \tag{16}$$

where $c\in\mathbb{R}$ is a constant.

In addition, $\sum_{i=1}^N y_i=2\pi$, $\forall t\ge 0$ always satisfies, with $\sum_{i=1}^N d_i=2\pi$, $y_i(t)=d_i\delta_i(t)$, we conclude that $c=1$. More precisely, $\lim_{t\to\infty}y(t)=d$ shows that the designed circle formation can be reached.

To avoid Zeno behaviour, an estimate of the positive lower bound on the inter-event times is further demonstrated. It is easy to obtain that for agent i, the event interval between t_{k+1}^i and t_k^i is the period that $\frac{\|e_i(t)\|}{\gamma_i\bar{\delta}_i}$ increases 0 to $\frac{\sigma}{\|\Upsilon L_d^T\|}$. Define $m=\arg\max_{i\in\mathcal{V}}\|\gamma_i\bar{\delta}_i\|$. Therefore, agent m stands for maximum the maximum norm of $\gamma_i\bar{\delta}_i$ among all the agents. It implies

$$\frac{\|e_m(t)\|}{\|\gamma_m\bar{\delta}_m\|}\le\frac{\|e(t)\|}{\|\gamma_m\bar{\delta}_m\|}\le\frac{\sqrt{N}\|e(t)\|}{\|\Upsilon\bar{\delta}\|}. \tag{17}$$

From (17), the time $\frac{\|e_m(t)\|}{\|\gamma_m\bar{\delta}_m\|}$ attains $\frac{\sigma}{\|\Upsilon L_d^T\|}$ is longer than $\frac{\sqrt{N}\|e(t)\|}{\|\Upsilon\bar{\delta}\|}$ costs. That is, we have $\tau_m>\tau$, where τ_m represents positive interval $(t_{k+1}^m-t_k^m)$ lower bound, τ is the time $\frac{\|e(t)\|}{\|\Upsilon\bar{\delta}\|}$ increases 0 to $\frac{\sigma}{\sqrt{N}\|\Upsilon L_d^T\|}$. Thereby, the time derivative of $\frac{\|e(t)\|}{\|\Upsilon\bar{\delta}\|}$ is

$$\begin{aligned}&\frac{d}{dt}\frac{\|e(t)\|}{\|\Upsilon\bar{\delta}\|}=\frac{d}{dt}\frac{(e(t)^Te(t))^{1/2}}{(\bar{\delta}^T\Upsilon\Upsilon\bar{\delta})^{1/2}}\\&=\frac{e(t)\dot{e}(t)}{\|e(t)\|\|\Upsilon\bar{\delta}\|}-\frac{\bar{\delta}^T\Upsilon\Upsilon\dot{\bar{\delta}}\|e(t)\|}{\|\Upsilon\bar{\delta}\|^3}\\&=\frac{-e(t)\Upsilon^{-1}\Upsilon(\bar{\delta}+L_d^Te(t))}{\|e(t)\|\|\Upsilon\bar{\delta}\|}-\frac{\bar{\delta}^T\Upsilon\Upsilon L_d^T(\bar{\delta}+L_d^Te(t))\|e(t)\|}{\|\Upsilon\bar{\delta}\|^2\|\Upsilon\bar{\delta}\|}\\&\le\frac{\|\Upsilon^{-1}\|(\|\Upsilon\bar{\delta}\|+\|\Upsilon L_d^Te(t))\|}{\|\Upsilon\bar{\delta}\|}+\frac{\|\Upsilon L_d^T\|\|\Upsilon^{-1}\|(\|\Upsilon\bar{\delta}\|+\|\Upsilon L_d^Te(t)\|)\|}{\|\Upsilon\bar{\delta}\|^2}\\&\le\|\Upsilon\|\left(1+\frac{\|e(t)\|\|\Upsilon L_d^T\|\|\Upsilon\bar{\delta}\|}{\|\Upsilon\bar{\delta}\|}\right)^2.\end{aligned} \tag{18}$$

Using β to stand for $\frac{\|e(t)\|}{\|\Upsilon\bar{\delta}\|}$, it yields $\dot{\beta}\le\|\Upsilon^{-1}\|(1+\|\Upsilon L_d^T\|\beta)^2$. Here, β satisfies the bound $\beta\le\alpha(t,\alpha_0)$, where $\alpha(t,\alpha_0)$ is the solution of $\dot{\alpha}(t,\alpha_0)=\|\Upsilon^{-1}\|(1+\|\Upsilon L_d^T\|\alpha(t,\alpha_0))^2$, $\alpha(0,\alpha_0)=\alpha_0$.

According to

$$\frac{d\alpha}{\|\Upsilon^{-1}\|(1+\|\Upsilon L_d^T\|\alpha(t,\alpha_0))^2}=dt, \tag{19}$$

We see that the interval between event instant t_k and t_{k+1} is lower bounded by the interval τ which satisfies $\alpha(\tau,0)=$

$\frac{\sigma}{\|\Upsilon L_d^T\|}$. By solving the difference equation (19), it yields

$$\begin{aligned}\tau &= \frac{d\alpha(\tau,0)}{\|\Upsilon^{-1}\|(1+\|\Upsilon L_d^T\|\alpha(\tau,0))} \\ &= \frac{\sigma}{(1+\sigma)\|\Upsilon L_d^T\|\|\Upsilon^{-1}\|}.\end{aligned} \tag{20}$$

From (20), we have

$$\tau = \frac{\sigma}{(\sqrt{N}+\sigma)\|\Upsilon L_d^T\|\|\Upsilon^{-1}\|}, \tag{21}$$

where τ is the time $\frac{\|e(t)\|}{\|\Upsilon\delta\|}$ from 0 to $\frac{\sigma}{\sqrt{N}\|\Upsilon L_d^T\|}$.

We observe that it leads to the minimal interval between two event instants of agent m

$$\tau_m = \frac{\sigma}{(\sqrt{N}+\sigma)\|\Upsilon L_d^T\|\|\Upsilon^{-1}\|} \tag{22}$$

It is easy to obtain $\tau_m > 0$, thus, we conclude that there exists at least one agents $m \in \mathcal{N}$ in system (10), which prevents occurrence of Zeno behavior under a observer-based event-trigger condition (12). □

IV. Simulation Results

Considering a MAS with 6 agents, the desired distances between each pair of adjacent agents is set to $d = \left[\frac{\pi}{8}, \frac{\pi}{2}, \frac{3\pi}{8}, \frac{\pi}{2}, \frac{\pi}{3}, \frac{\pi}{6}\right]^T$ which meet (6), and the initial values of MAS are randomly generated satisfying (5). Additionally, the unique normalized positive left eigenvector of L_d^T with respect to eigenvalue 0 is $\xi = [0.0625\ 0.0.25\ 0.1875\ 0.25\ 0.1667\ 0.0833]^T$. Note that all of the simulated event detections are implemented as sample data. Thus, the sampling periods h in real-time control is set to $0.2s$.

By the permitted range $0 < \sigma < 1$, we set $\sigma = 0.9$ to ensure the condition (12) hold in real-time control. Figure 3 (a) shows the evolution difference between the event-triggered angular distance and the expected counterpart, Figure 3 (b) reveals the event sequence of each agent. Figure 4 illustrates the fluttering of the measurement error $\|y_i(t_k^i) - y_i(t)\|$.

We can obtain from the simulation results that the desired circle formation can be asymptotically solved by the proposed control scheme for distributed first-order MASs. We also calculate the average inter-event time overall mobile agents $h_{avg} = 0.6059$ from Figure 3, the result indicates that our method can reduce the amount of control update for formation control of MASs.

V. Conclusion

This paper investigated the circle formation control problem for first-order MASs under unbalanced directed networks with limited resource constraints. We first designed the observer-based event-triggered algorithm to reduce dependence on resources, in which, when the value of the event-triggering condition exceeds zero, the agent's controller will update the agent's states simultaneously. Moreover, it is a fundamental and practical aspect to observe neighbor information on a regular or better basis continuously. Then, we proved that if there is a spanning tree in the underlying graph, the MASs can achieve the desired circle formation by the proposed control laws, and Zeno behaviors can be ruled out. In the end, we gave numerical simulation examples to show that for first-order MASs, the proposed event-triggered circle formation control strategies are effective. In future work, we will extend our research to more practical operations, e.g., considering the effect of time delays in communication networks, input saturation constraints, and weak links.

References

[1] A. Macwan, J. Vilela, G. Nejat, and B. Benhabib, "Multi-robot deployment for wilderness search and rescue," *Int. J. Robot. Autom.*, vol. 31, no. 1, 2016.

[2] K. Kim, H. Kim, and H. Myung, "Bio-inspired robot swarm control algorithm for dynamic environment monitoring," *Advances in robotics research*, vol. 2, no. 1, pp. 1–11, 2018.

[3] T. Yang, Z. Meng, D. V. Dimarogonas, and K. H. Johansson, "Global consensus for discrete-time multi-agent systems with input saturation constraints," *Automatica*, vol. 50, no. 2, pp. 499–506, 2014.

[4] P. Lin, W. Ren, and Y. Song, "Distributed multi-agent optimization subject to nonidentical constraints and communication delays," *Automatica*, vol. 65, pp. 120–131, 2016.

[5] V. R. Desaraju and J. P. How, "Decentralized path planning for multi-agent teams with complex constraints," *Autonomous Robots*, vol. 32, no. 4, pp. 385–403, 2012.

[6] S. Mastellone, D. M. Stipanović, C. R. Graunke, K. A. Intlekofer, and M. W. Spong, "Formation control and collision avoidance for multi-agent non-holonomic systems: Theory and experiments," *The International Journal of Robotics Research*, vol. 27, no. 1, pp. 107–126, 2008.

[7] C. Wang, G. Xie, and M. Cao, "Forming circle formations of anonymous mobile agents with order preservation," *IEEE Transactions on Automatic Control*, vol. 58, no. 12, pp. 3248–3254, 2013.

[8] K.-K. Oh, M.-C. Park, and H.-S. Ahn, "A survey of multi-agent formation control," *Automatica*, vol. 53, pp. 424–440, 2015.

[9] Z. Bu, Z. Wu, J. Cao, and Y. Jiang, "Local community mining on distributed and dynamic networks from a multiagent perspective," *IEEE Transactions on Cybernetics*, vol. 46, no. 4, pp. 986–999, 2017.

[10] D. Tian, J. Zhou, and Z. Sheng, "An adaptive fusion strategy for distributed information estimation over cooperative multi-agent networks," *IEEE Transactions on Information Theory*, vol. 63, no. 5, pp. 3076–3091, 2017.

[11] G. Wen, H.-T. Zhang, W. Yu, Z. Zuo, and Y. Zhao, "Coordination tracking of multi-agent dynamical systems with general linear node dynamics," *International Journal of Robust and Nonlinear Control*, vol. 27, no. 9, pp. 1526–1546, 2017.

[12] R. Olfati-Saber and R. M. Murray, "Consensus problems in networks of agents with switching topology and time-delays," *IEEE Transactions on Automatic Control*, vol. 49, no. 9, pp. 1520–1533, 2004.

[13] W. Xie, B. Ma, T. Fernando, and H. C. Iu, "A new formation control of multiple underactuated surface vessels," *International Journal of Control*, vol. 91, no. 10, pp. 1–12, 2017.

[14] P. Tabuada, "Event-triggered real-time scheduling of stabilizing control tasks," *IEEE Transactions on Automatic Control*, vol. 52, no. 9, pp. 1680–1685, 2007.

[15] G. S. Seyboth, D. V. Dimarogonas, and K. H. Johansson, "Event-based broadcasting for multi-agent average consensus," *Automatica*, vol. 49, no. 1, pp. 245–252, 2013.

[16] D. Liuzza, D. V. Dimarogonas, M. Di Bernardo, and K. H. Johansson, "Distributed model based event-triggered control for synchronization of multi-agent systems," *Automatica*, vol. 73, pp. 1–7, 2016.

[17] W. Heemels, K. H. Johansson, and P. Tabuada, "An introduction to event-triggered and self-triggered control," in *2012 IEEE 51st IEEE Conference on Decision and Control (CDC)*. IEEE, 2012, pp. 3270–3285.

[18] D. V. Dimarogonas, E. Frazzoli, and K. H. Johansson, "Distributed event-triggered control for multi-agent systems," *IEEE Transactions on Automatic Control*, vol. 57, no. 5, pp. 1291–1297, 2011.

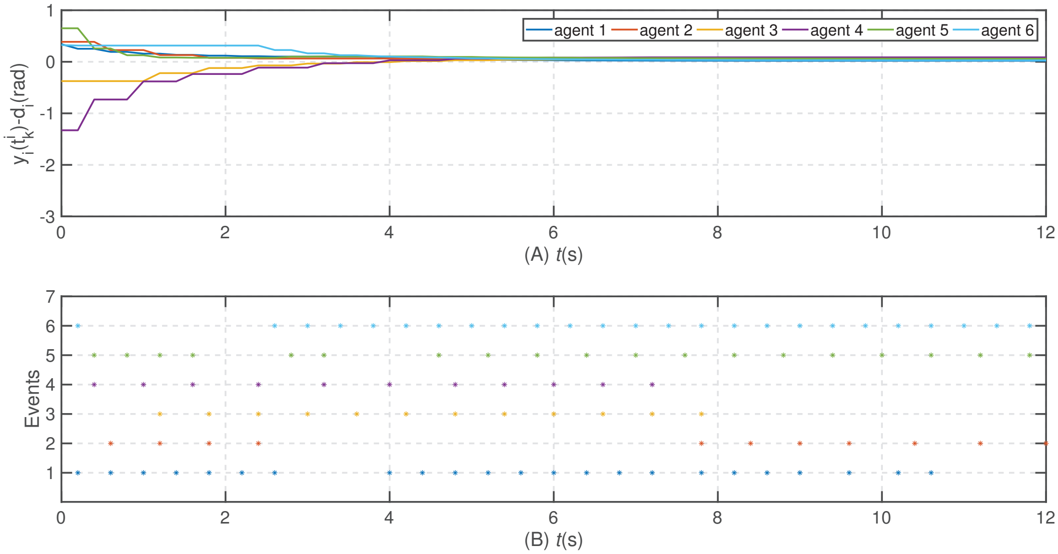

Fig. 3. The evolution of $y_i(t_k^i) - d_i$, for $i = 1, 2, ..., N$ when $h = 0.2s$.

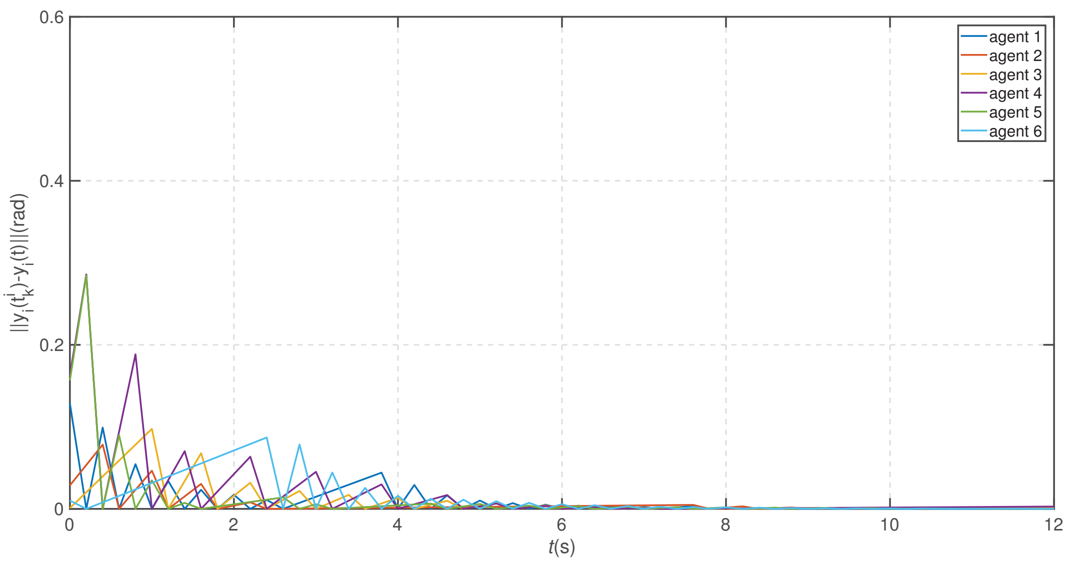

Fig. 4. The evolution of $\|y_i(t_k^i) - y_i(t)\|$ for $i = 1, 2, ..., N$ when $h = 0.2s$.

[19] J. Wen, C. Wang, and G. Xie, "Asynchronous distributed event-triggered circle formation of multi-agent systems," *Neurocomputing*, vol. 295, pp. 118–126, 2018.

[20] J. Wen, P. Xu, C. Wang, G. Xie, and Y. Gao, "Distributed event-triggered circle formation control for multi-agent systems with limited communication bandwidth," *Neurocomputing*, vol. 358, pp. 211–221, 2019.

[21] P. Xu, J. Wen, C. Wang, and G. Xie, "Distributed circle formation control over directed networks with communication constraints," *IFAC-PapersOnLine*, vol. 52, no. 3, pp. 108–113, 2019.

[22] K. H. Johansson, M. Egerstedt, J. Lygeros, and S. Sastry, "On the regularization of zeno hybrid automata," *Systems Control Lett*, vol. 38, no. 3, pp. 141–150, 1999.

[23] M. Heymann, F. Lin, G. Meyer, and S. Resmerita, "Analysis of zeno behaviors in a class of hybrid systems," *IEEE Transactions on Automatic Control*, vol. 50, no. 3, pp. 376–383, 2005.

[24] C. Nowzari, *Distributed event-triggered coordination for average consensus on weight-balanced digraphs*. Pergamon Press, Inc., 2016.

[25] S. Zaman, W. Slany, and G. Steinbauer, "Ros-based mapping, localization and autonomous navigation using a pioneer 3-dx robot and their relevant issues," in *Electronics, Communications and Photonics Conference*, 2011, pp. 1–5.

[26] C. Wang and G. Xie, "Limit-cycle-based decoupled design of circle formation control with collision avoidance for anonymous agents in a plane," *IEEE Transactions on Automatic Control*, vol. 62, no. 12, pp. 6560–6567, 2017.

[27] H. J. Brascamp and E. H. Lieb, "Best constants in young's inequality, its converse, and its generalization to more than three functions," *Advances in Mathematics*, vol. 20, no. 2, pp. 151–173, 1976.
[28] W. Ren and R. W. Beard, "Consensus seeking in multiagent systems under dynamically changing interaction topologies," *IEEE Transactions on Automatic Control*, vol. 50, no. 5, pp. 655–661, 2005.
[29] D. Xie, S. Xu, Z. Li, and Y. Zou, "Event-triggered consensus control for second-order multi-agent systems," *IET Control Theory & Applications*, vol. 9, no. 5, pp. 667–680, 2015.