

Distributed Self-triggered Circular Formation Control for Multi-robot Systems

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Abstract: This paper investigates circular formation control problems for multi-robot systems in the plane via a distributed self-triggered strategy. In scenarios of restricted energies, a distributed self-triggered protocol is designed for controlling multiple robots to converge asymptotically to a prescribed circular orbit around a fixed target. In particular, each robot maintains any desired relative angular distances during its rotation around the target. Besides, no collision among robots is taken place, since the spatial order of robots is preserved throughout the evolution. We prove that when the event-triggered condition is enforced during the whole process, the controllers only update with superior performance. Moreover, Zeno behavior can be ruled out. Numerical simulations demonstrate the feasibility and effectiveness of the theoretical results.

Key Words: Multi-robot Systems, Circular Formation, Self-triggered, Directed Network

1 Introduction

A multi-robot system (MRS) is a group of interconnected robots, which are embedded with certain sense, communication, and actuation capabilities. In the current research, special attention is paid to the design, analysis, and implementation of coordinated collective control [1–4]. Notably, formation control, as a most actively studied topic in coordination control, aims to drive all robots to form and maintain a predetermined geometry [5–8]. However, from a practical viewpoint, robots are allowed to fulfill complex tasks with higher energy consumption, such as source localization [9, 10] and surveillance [11–13]. Therefore, it is desirable to design algorithms that require efficient use of the throughput capacity of the communication bandwidth, as well as the computational resources of the device.

Motivated by the above consideration, event-triggered control (ETC) mechanism is introduced to reduce the costs of computation, communication, and actuator effectively without degrading performance [14–19]. The most significant feature of ETC is that when the norm of a function with the state exceeds the trigger threshold of specific measurement error, control actions and communications are only enforced throughout the evolution of the system. Intensive research efforts have been devoted to distributed formation control for MRS via ETC strategies. In [14], both centralized and distributed ETC algorithms were designed to achieve consensus concerning a first-order multi-agent system. Together with encoder-decoder strategies in [20, 21], an innovative algorithm was investigated to address the circle formation problem with a limitation of communication, which depends on the combination between the quantized commu-

nication technology and ETC. Moreover, as to second-order MRSs, a distributed ETC algorithm was proposed to reach consensus in [16]. Along this line, for further reduction in computation and transmission cost [17], sampled data, self-triggered scheme, and data quantization was employed together to guarantee the quantized consensus of second-order MRS. In most previous works [16, 17, 20, 22, 23], motions of all robots are restricted to a 1-D space, which is inadequate to extend directly to the motions of robots in the plane.

Thus, as natural choices of the geometric shapes to perform complex tasks, circular formation for MRSs in the plane has received the special attention of researchers. In general, circular formation problems can be classified into two major tasks [24], one is target circling, which aims to drive all robots to converge onto the circle around the target; the other is a spacing adjustment, which refers to adjusting all robots to reach the desired angle distance between pairs of neighboring. In [25], a limit-cycle-based decoupled-design approach was proposed to the circular formation problem, where each robot is modeled as a kinematic point and can merely obtain the relative positions of the target and its limited neighbors. However, so far as we know, there have been no systematic investigations on the circular formation with ETC in the plane.

Different from previous works, especially in [20, 21, 25], the main goal of this paper is to study the circular formation for MRSs via self-triggered strategies in the 2-D space. In fact, the self-triggered strategy is regarded as a special case of the ETC. In our MRS, each robot only perceives relative positions of the target and the distance from itself to its nearest neighbor in a counterclockwise direction, and the counterpart in the clockwise direction through communication [26]. The main contributions of this paper are listed as follows. Firstly, based on the additional states in the protocol designed, an alternative distributed self-triggered algorithm is proposed to reduce the computation and communication between neighbors. Secondly, a Lyapunov function

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approach is utilized to check the asymptotic stability of MRS under the self-triggered mechanism. Moreover, sufficient conditions for the absence of Zeno behavior are derived. At last, simulation results are demonstrated that the designed algorithm can drive all mobile robots to form expected circular formations in 2-D space.

The remainder of this paper is organized as follows. In Section 2, preliminary definitions, and the problem formulation are given. Section 3 addresses a self-triggered circle formation problem without continuous monitoring of the state of neighbors. Simulation results are given in Section 4 to validate the theoretical analysis. Section 5 draws a conclusion and indicates possible extensions.

2 Preliminaries and problem statement

In this section, we first give some notions and basic concepts from algebraic graph theory, then formulate the circular formation problem in the plane for MRS.

2.1 Preliminaries

\mathbb{R} and I_N are the set of real numbers and $N \times N$ identity matrix, respectively. For a vector or a matrix A , its Euclidean norm is denoted by $\|A\|$ and its transpose is denoted by A^T . Vectors $\mathbf{1}_N$ and $\mathbf{0}_N$ denote N dimension column vectors with all entries equal to 1 and 0, respectively. $\text{diag}\{a_1, a_2, \dots, a_N\}$ represents the diagonal matrix with diagonal elements a_1, a_2, \dots, a_N .

The following lemmas are used in our theoretical analysis.

Lemma 1 ([27]) *For any $x, y \in \mathbb{R}$ and $a > 0$, the following properties are applied*

$$\begin{aligned} 1. xy &\leq \frac{a}{2}x^2 + \frac{1}{2a}y^2; \\ 2. (x^2 + y^2) &\leq (x + y)^2, \text{ if } xy \geq 0. \end{aligned} \quad (1)$$

Lemma 2 ([28]) *Given a directed graph \mathcal{G} , composed of a spanning tree, the vector $\xi = [\xi_1, \xi_2, \dots, \xi_N]^T > 0$ satisfies $\sum_{i=1}^N \xi_i = 1$ and $\xi^T \mathcal{L} = \mathbf{0}_N$, in which ξ denotes the left eigenvector corresponding to zero eigenvalue of the Laplacian matrix \mathcal{L} . Furthermore, $\mathcal{L}^T \Theta + \Theta \mathcal{L}^T$ is semi-positive definite where $\Theta = \text{diag}\{\xi_1, \xi_2, \dots, \xi_N\}$. After take square root of each element of Θ , we obtain $\Upsilon = \text{diag}\{\gamma_1, \gamma_2, \dots, \gamma_N\}$. Consequently, $\gamma = \sqrt{\xi}, i = 1, \dots, N$.*

2.2 Problem formulation

Suppose an MRS in an obstacle-free plane, consisting of N mobile robots $P = (p_1, p_2, \dots, p_N)$ and a prescribed target p_0 that to be circular around, as shown in Figure 1. Here, each robot is anonymous that cannot recognize the owner of the token, but has a single marker available that can be left on a single whiteboard. The initial positions of robots are randomly generated, whereas no two robots occupy the same position as the target simultaneously. For simplicity, the robots are labeled based on their initial positions according to the following three rules [25].

- 1) The labels are sorted in ascending order counterclockwise around the target.
- 2) For a robot located on the same ray extending from the target, its label is sorted in ascending order from the distance to the target point.
- 3) For robots occupying the same position, their labels will be randomly selected.

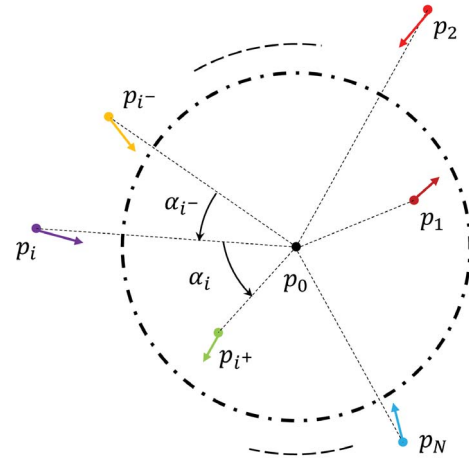


Fig. 1: N robots are initially located in a plane.

Then, we consider a case that robots' neighbor relationships are modeled by a directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$, where $\mathcal{V} = \{p_1, p_2, \dots, p_N\}$ denotes a group of mobile robots, $\mathcal{E} = \mathcal{V} \times \mathcal{V}$ is a set communication edges connecting pairs of robots, and $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N \times N}$ denotes a weighted adjacency matrix. In this relationship, each robot has only two adjacent neighbors, i.e., in front of or behind itself, marked as $\mathcal{N}_i = \{i^-, i^+\}$, where

$$i^+ = \begin{cases} i + 1, & i = 1, 2, \dots, N - 1, \\ 1, & i = N, \end{cases} \quad (2)$$

and

$$i^- = \begin{cases} N, & i = 1, \\ i - 1, & i = 2, 3, \dots, N. \end{cases} \quad (3)$$

Let $p_i(t) = [x_i(t), y_i(t)]^T \in \mathbb{R}^2$ be the position of robot p_i at time t , and $p_0 = [x_0, y_0]^T \in \mathbb{R}^2$ stands for predefined target point. Therefore, each robot is modeled by a kinematic point

$$\dot{p}_i(t) = u_i(t), \quad i = 1, 2, \dots, N, \quad (4)$$

where $u_i(t) \in \mathbb{R}^2$ is the control input of robot p_i to be designed.

Suppose that each robot can only use the relative positions between the target and its two neighbors under the neighbor relationship \mathcal{G} . Note that by sensing and using the information about its two immediate neighbors, the robots do not need to know the label information. The following variables are introduced to formulate the problem. Let

$$\hat{p}_i(t) = p_i(t) - p_0, \quad i = 1, 2, \dots, N, \quad (5)$$

be the position of robot p_i relative to target p_0 .

The position of robot p_i relative to its neighbor p_{i^+} is expressed as

$$\tilde{p}_i(t) = p_{i^+} - p_i(t), \quad i = 1, 2, \dots, N. \quad (6)$$

The angle between robot p_i and robot p_{i^+} is described as

$$\alpha_i(t) = \angle p_i p_0 p_{i^+}. \quad (7)$$

Similarly, as shown in Figure 2, let $\alpha_{i^-}(t)$ denote the angle between robot p_i and robot p_{i^-} . Then, α_i^* represents the

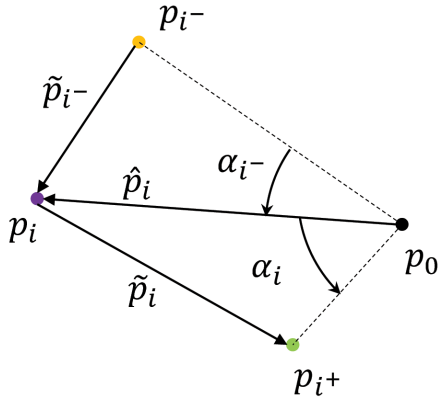


Fig. 2: Locally implementable control.

desired angle spacing from robot p_i to robot p_{i+} , such that the desired angle spacing of N robots is determined by the vector

$$\alpha^* = [\alpha_1^*, \alpha_2^*, \dots, \alpha_N^*]^T. \quad (8)$$

It is worth noting that there exists $r > 0$, $\alpha_i^* > 0$ and $\sum_{i=1}^N \alpha_i^* = 2\pi$ such that a desired circular formation is admissible, where r is the desired radius of the circle.

Above all, in order to provide the N anonymous robots initial states with their labels combined with the mathematical descriptions, the following definitions of the robots spatial ordering are proposed.

Definition 1 (Counterclockwise Order) The N robots are indicated to be arranged in a counterclockwise order if $\alpha_i \in (0, 2\pi)$ for all $i = 1, 2, \dots, N$ and $\sum_{i=1}^N \alpha_i = 2\pi$.

Definition 2 (Almost Counterclockwise Order) The N robots are indicated to be arranged in an almost counterclockwise order if 1) $\alpha_i \in [0, 2\pi)$ for all $i = 1, 2, \dots, N$ and $\sum_{i=1}^N \alpha_i = 2\pi$; and 2) $\alpha_i = 0$ means $\|\hat{p}_{i+}\| > \|\hat{p}_i\|$, where $\|\cdot\|$ denotes the Euclidean norm.

Below, the definition of circular formation problem is described as follows.

Definition 3 (Circular Formation Problem) Given an admissible circular formation in the plane characterized by α^* and r , a distributed control protocol $u_i(t, r, \hat{p}_i(t), \alpha_i(t))$, $i = 1, 2, \dots, N$ is designed such that the solution to MRS (4) converges to the equilibrium point P under any initial condition, namely,

$$\|\hat{P}_i\| = r, \quad i = 1, 2, \dots, N, \text{ (Target radius)} \quad (9)$$

and

$$\alpha = \alpha^*, \quad \text{(Spacing adjustment)} \quad (10)$$

are satisfied.

Moreover, required properties of MRSs have been proposed as follows.

Definition 4 (Order Preservation) For a N -robots system, it is able to obtain that under the control protocol $u_i(t)$, the robots' spatial ordering is maintained if N robots initially locates in an almost counterclockwise order in the plane.

The solution to MRS (4) can guarantee N robots maintain in a counterclockwise order for all $t > 0$.

Definition 5 (Collision Avoidance) For a N -robots system, we claim that under the control protocol $u_i(t)$, the robots have the property of collision avoidance if N robots are initially arranged in an almost counterclockwise order in the plane. The solution to MRS (4) satisfies $\|p_i\| - \|p_j\| > 0$ for any pair of i, j ($i \neq j$) for all $t > 0$.

3 Self-triggered control strategy

Given a sampled-date protocol designed in [25], given as

$$u_i(t) = \varphi \begin{bmatrix} k_r l_i(t) & -1 \\ 1 & k_r l_i(t) \end{bmatrix} \hat{p}_i(t) g_i(t), \quad i = 1, 2, \dots, N, \quad (11)$$

where $l_i(t) = r^2 - \|\hat{p}_i(t)\|$, $\varphi > 0$, $k_r > 0$ are constant, and

$$g_i(t) = 1 + \frac{1}{2\pi} \left[\frac{\alpha_{i-}^*}{\alpha_i^* + \alpha_{i-}^*} \alpha_i(t) - \frac{\alpha_i^*}{\alpha_i^* + \alpha_{i-}^*} \alpha_{i-}(t) \right]. \quad (12)$$

From (12), the variable α_i can be treated as an additional state of the MRS. Evidently, each robot has to transmit a request continuously to its neighbors for acquiring their additional states, and then calculate the value of $g_i(t)$ and $l_i(t)$ using processing chip. However, robots usually have limitations of communication and computation capabilities, which caused that control protocol (11) can not be applied in practice.

In order to address this issue, a self-triggered strategy relying on the additional state is proposed, in which communication between the pairs of neighboring and computation is only enforced at discrete event instants. Thus, undesirable transmission and computation are efficiently avoided. Below, let an increasing sequence $(t_0^i, t_1^i, \dots, t_k^i, \dots)$ denote the event instants of robot p_i , such that $\alpha_i(t_k^i)$ is the state of of robot p_i at the k -th event instants. Note that due to all robots trigger asynchronously, each robot has its own event sequence. Then, the control law coupled with the self-triggered scheme is designed as

$$u_i(t) = \varphi \begin{bmatrix} k_r l_i(t_k^i) & -1 \\ 1 & k_r l_i(t_k^i) \end{bmatrix} \hat{p}_i(t) g_i(t_k^i), \quad t \in (t_k^i, t_{i}^{k+1}]. \quad (13)$$

Substituting (13) into (4), the resulting closed-loop dynamics of robot p_i can be rewritten as

$$\dot{p}_i(t) = \varphi \begin{bmatrix} k_r l_i(t_k^i) & -1 \\ 1 & k_r l_i(t_k^i) \end{bmatrix} \hat{p}_i(t) g_i(t_k^i), \quad i = 1, 2, \dots, N. \quad (14)$$

By $\hat{p}_i(t)$, (14) can be rearranged as

$$\dot{\hat{p}}_i(t) = \varphi \begin{bmatrix} k_r l_i(t_k^i) & -1 \\ 1 & k_r l_i(t_k^i) \end{bmatrix} \hat{p}_i(t) g_i(t_k^i), \quad i = 1, 2, \dots, N. \quad (15)$$

Furthermore, using (7), we have

$$\dot{\alpha}_i(t_k^i) = \dot{\alpha}_{i+}(t_k^i) - \dot{\alpha}_i(t_k^i), \quad i = 1, 2, \dots, N, \quad (16)$$

where $\tilde{\alpha}_i(t_k^i)$ denotes the angle of the vector $\hat{p}_i(t_k^i)$.

Then,

$$\begin{aligned} \dot{\tilde{\alpha}}_i(t_k^i) &= \varphi g_i(t_k^i), \\ \|\dot{\hat{p}}_i(t_k^i)\| &= k_r \varphi \|\hat{p}_i(t_k^i)\| (r^2 - \|\hat{p}_i(t_k^i)\|^2) g_i(t_k^i), \end{aligned} \quad (17)$$

Substituting (17) into (16), the dynamical equation of the additional states combined with the self-triggered strategy is obtained as

$$\dot{\alpha}_i(t) = \varphi (g_{i+}(t_k^i) - g_i(t_k^i)), t \in [t_k^i, t_{k+1}^i], \quad (18)$$

For simplicity, assume that $\hat{\alpha}_i(t) = \alpha_i(t_k^i)$, $\hat{\delta}_i(t) = \hat{\alpha}_i(t)/\alpha_i^*$, (18) can be represented as

$$\begin{aligned} \alpha_i^* \dot{\delta}_i(t) &= \frac{\varphi}{2\pi} \left(\left[\frac{\alpha_i^*}{\alpha_{i+}^* + \alpha_i^*} \hat{\alpha}_{i+}(t) - \frac{\alpha_{i+}^*}{\alpha_{i+}^* + \alpha_i^*} \hat{\alpha}_i(t) \right] - \right. \\ &\quad \left. \left[\frac{\alpha_i^*}{\alpha_i^* + \alpha_{i-}^*} \hat{\alpha}_i(t) - \frac{\alpha_{i-}^*}{\alpha_i^* + \alpha_{i-}^*} \hat{\alpha}_{i-}(t) \right] \right). \end{aligned} \quad (19)$$

Using δ_i , (19) can be summarized into a simple form as

$$\dot{\delta}_i(t) = \frac{\varphi}{2\pi} \sum_{j \in \mathcal{N}_i} \frac{\alpha_j^*}{\alpha_i^* + \alpha_j^*} (\hat{\delta}_j(t) - \hat{\delta}_i(t)), t \geq 0. \quad (20)$$

A deviation variable is defined as $e_i(t) = \hat{\delta}_i(t) - \delta_i(t)$. Then a compact form of the system dynamics can be derived as

$$\dot{\delta}(t) = -\frac{\varphi}{2\pi} L_d^T (\delta(t) + e(t)), t \in [t_k^i, t_{k+1}^i], \quad (21)$$

where $\delta(t) = [\delta_1(t), \delta_2(t), \dots, \delta_N(t)] \in \mathbb{R}^N$, and $e(t) = [e_1(t), e_2(t), \dots, e_N(t)] \in \mathbb{R}^N$.

For the designed α_i ' dynamical equation (18), the self-triggered circular formation control for distributed cases can be solved in the following theorem.

Theorem 1 *Given any admissible circular formation characterized by α^* and r , and considering an MRS (4) and the designed control law (13) over a strongly connected weight-unbalanced digraph \mathcal{G} , the Circular Formation Problem is solvable when the event-trigger condition is designed as*

$$\tilde{f}_i(t) = \|e_i(t)\| - \frac{\|L_d^T(i, j)\hat{\delta}(t)\|}{(b+1)\|L_d^T\|\rho e^{\|l_i(t)\|}}, b > 0, \quad (23)$$

where $L_d^T(i, j)\hat{\delta}(t) = \sum_{j \in \mathcal{N}_i} L_d^T(i, j)(\hat{\delta}_i(t) - \hat{\delta}_j(t))$. And the condition

$$-\xi_i + \frac{\xi_i^2}{2a} + \frac{b+1}{b^3(t)M} > 0, i = 1, 2, \dots, N, \quad (24)$$

holds simultaneously, where $M = \min\{\|\rho e^{\|l_i(t)\|}\}$. Moreover, the self-trigger condition (23) helps the MRS to avoid occurrence of Zeno behavior.

Proof:

A Lyapunov function candidate is considered as

$$V(t) = \frac{1}{4} \delta^T(t) (L_d \Theta + \Theta L_d^T) \delta(t), \quad (25)$$

where Θ is the same diagonal matrix as described in Lemma 2 such that $L_d \Theta + \Theta L_d^T$ is semi-positive definite. As a result, $V(t) > 0$ and $\dot{V}(t) = 0$ if and only if the Circular Formation Problem is solvable.

Then the derivative of the Lyapunov function along of the trajectories of MRS (4) yields to

$$\begin{aligned} \dot{V}(t) &= \delta^T(t) L_d \Theta \left(-\frac{\varphi}{2\pi} L_d^T (\delta(t) + e(t)) \right) \\ &= -\frac{\varphi}{2\pi} \delta^T(t) L_d \Theta L_d^T \delta(t) - \frac{\varphi}{2\pi} \delta^T(t) L_d \Theta L_d^T e(t). \end{aligned} \quad (26)$$

From Lemma 1, there exists $\delta^T(t) L_d \Theta L_d^T e(t) \leq \frac{1}{2a} \delta^T(t) L_d \Theta^2 L_d^T \delta(t) + \frac{a}{2} e^T(t) L_d L_d^T e(t)$ such that (26) is rearranged into

$$\begin{aligned} \dot{V}(t) &\leq -\frac{\varphi}{2\pi} \delta^T(t) L_d \Theta L_d^T \delta(t) + \\ &\quad \frac{\varphi}{2\pi} \left(\frac{1}{2a} \delta^T(t) L_d \Theta^2 L_d^T \delta(t) + \frac{a}{2} e^T(t) L_d L_d^T e(t) \right). \end{aligned} \quad (27)$$

In the following, we explain the analytical relationship between $\delta^T(t) L_d \Theta L_d^T \delta(t)$ and $e^T(t) L_d L_d^T e(t)$.

From the self-trigger condition (23), we have

$$\|L_d^T e(t)\| \leq \|L_d^T\| \|e(t)\| \leq \frac{\|L_d^T \hat{\delta}(t)\|}{(b+1)\rho e^{\|l_i(t)\|}}. \quad (28)$$

Together with the definition of e_i and (28), it yields to

$$\begin{aligned} e^T(t) L_d L_d^T e(t) &\leq \frac{1}{(b+1)^2 \rho^2 e^{2\|l_i(t)\|}} \\ &\quad ((\delta(t) + e(t))^T L_d L_d^T (\delta(t) + e(t))) \\ &\leq \frac{1}{(b+1)^2 \rho^2 e^{2\|l_i(t)\|}} (\delta(t)^T L_d L_d^T \delta(t) + \\ &\quad (.e(t)^T L_d L_d^T e(t) + 2\delta(t)^T L_d L_d^T e(t))) \\ &\leq \frac{1}{(b+1)^2 \rho^2 e^{\|l_i(t)\|}} \left(1 + \frac{1}{b}\right) \delta(t)^T L_d L_d^T \delta(t) \\ &\quad + \frac{1+2b}{(b+1)^2 \rho^2 e^{2\|l_i(t)\|}} e(t)^T L_d L_d^T e(t). \end{aligned} \quad (29)$$

Thus,

$$e^T(t) L_d L_d^T e(t) \leq \frac{b+1}{b^3(t)M} \delta(t)^T L_d L_d^T \delta(t). \quad (30)$$

Substituting (30) into (27), we have

$$\begin{aligned} \dot{V}(t) &\leq -\frac{\varphi}{2\pi} \delta^T(t) L_d \Theta L_d^T \delta(t) + \frac{\varphi}{2\pi} \left(\frac{1}{2a} \delta^T(t) L_d \Theta^2 L_d^T \delta(t) \right. \\ &\quad \left. + \frac{b+1}{b^3(t)M} \delta(t)^T L_d L_d^T \delta(t) \right) \\ &\leq -\frac{\varphi}{2\pi} \sum_{i=1}^N \left(-\xi_i + \frac{\xi_i^2}{2a} + \frac{b+1}{b^3(t)M} \right) \|\bar{\delta}_i\|. \end{aligned} \quad (31)$$

Therefore, the condition (23) guarantees $\dot{V}(t) < 0$ and $\dot{V}(t) = 0$ if and only if the Circular Formation Problem is solvable.

$$L_d = \begin{bmatrix} \frac{\alpha_2^*}{\alpha_2^* + \alpha_1^*} + \frac{\alpha_N^*}{\alpha_N^* + \alpha_1^*} & -\frac{\alpha_1^*}{\alpha_2^* + \alpha_1^*} & 0 & \dots & 0 & -\frac{\alpha_1^*}{\alpha_N^* + \alpha_1^*} \\ -\frac{\alpha_2^*}{\alpha_2^* + \alpha_1^*} & \frac{\alpha_3^*}{\alpha_3^* + \alpha_2^*} + \frac{\alpha_1^*}{\alpha_2^* + \alpha_1^*} & -\frac{\alpha_2^*}{\alpha_3^* + \alpha_2^*} & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & \frac{\alpha_N^*}{\alpha_N^* + \alpha_{N-1}^*} + \frac{\alpha_{N-2}^*}{\alpha_{N-1}^* + \alpha_{N-2}^*} & -\frac{\alpha_{N-1}^*}{\alpha_N^* + \alpha_{N-1}^*} \\ -\frac{\alpha_N^*}{\alpha_N^* + \alpha_1^*} & 0 & 0 & \dots & -\frac{\alpha_N^*}{\alpha_N^* + \alpha_{N-1}^*} & \frac{\alpha_1^*}{\alpha_N^* + \alpha_1^*} + \frac{\alpha_{N-1}^*}{\alpha_N^* + \alpha_{N-1}^*} \end{bmatrix} \quad (22)$$

To avoid Zeno behaviour, an estimate of the positive lower bound on the inter-event times is further proved. Assuming that the $k + 1$ th event of robot p_i occurs at the time $t_k^i + \tau_i$, we derive $\|e_i(t_k^i)\| = 0$, and

$$\|e_i(t_k^i + \tau_i)\| = \frac{\|L_d^T(i, j)\hat{\delta}(t)\|}{(b+1)\|L_d^T\|\rho e^{\|l_i(t)\|}}.$$

From the trajectory of $e_i(t)$, we have

$$\begin{aligned} \|e_i(t_k^i + \tau_i)\| &= \left\| \int_{t_k^i}^{t_k^i + \tau_i} \dot{e}_i(t) dt \right\| \\ &= \left\| \int_{t_k^i}^{t_k^i + \tau_i} \dot{\delta}_i(t) dt \right\| \\ &= \left\| \int_{t_k^i}^{t_k^i + \tau_i} \frac{\varphi}{2\pi} L_d^T(i, j)\hat{\delta}(t) dt \right\| \\ &\leq \frac{\varphi}{2\pi} \|L_d^T(i, j)\hat{\delta}(t)\| \tau \end{aligned} \quad (32)$$

Substituting (23) into (32), we get

$$\frac{\|L_d^T(i, j)\hat{\delta}(t)\|}{(b+1)\|L_d^T\|\rho e^{\|l_i(t)\|}} \leq \frac{\varphi}{2\pi} \|L_d^T(i, j)\hat{\delta}(t)\| \tau, \quad (33)$$

where $\tau = \frac{2\pi}{\varphi(b+1)\|L_d^T\|\rho e^{\|l_i(t)\|}} \geq 0$.

To sum up, if a neighbour triggers during the interval between two consecutive events of robot p_i , that is, the neighbour triggers at time $t_k^i + \tau_j \leq t_k^i + \tau_i$. Then the interval is greater than τ_j . We can conclude that the intervals between events that generated by the self-triggered function are positive.

4 Simulation Results

Considering an MRS, consists of six mobile robots locating in a plane. The target point is set to $(0, 0)$, and the desired radius of the circular and the desired angle distances between each pair of neighboring robots are set to satisfy (8). The initial positions of the six robots are randomly generated. To show the relative superiority of the self-triggered strategy, the event detection of all those simulations is executed using a sampled-data approach. Here, $h = 0.01s$ is chosen as the sampling periods in real-time control. Coefficients of the controller are set to $\varphi = 0.0002$, $k_r = 0.4$, so as to make ensure the condition (11) holds. In addition, the role of coefficients mentioned is to keep $l_i(t)$ remain at least an order of magnitude comparing to $g(t)$.

We first apply the self-triggered control strategy to the uniform circular formation control with the desired angle distance $\alpha_i^* = \pi/3$ and the desired radius of the circular $r = 100$. We run the simulation and show the results of

the proposed control laws solving the uniform circular formation problem with the self-triggered function (23). Figure (3) reveals the trajectories of six robots in the plane at $t \in [0, 200]$, respectively. Figure (4) shows the difference between the self-triggered angle distance and the expected counterpart, the distances between the self-triggered radius of the circular and the prescribed radius, and the evolution of control laws for all the six robots, respectively. Moreover, the average inter-event time for all robots is obtained as $h_{avg} = 0.0229$. It is obvious that trade-offs among communication and computation would be reduced dramatically by as much as $1/3$ due to the application of the self-triggered strategy.

Furthermore, we extend the method to the non-uniform circular formation, where the desired angle distance is set to $\alpha^* = [\pi/4, \pi/3, 3\pi/8, 7\pi/24, \pi/3, 5\pi/12]$. Moreover, the same r and initial positions of robots in the first case are utilized. The simulation results of the proposed control laws solving the non-uniform circular formation problem with the self-triggered function (23) are shown in Figure (5)-(6). Similarly, the average inter-event time for all robots is obtained as $h_{avg} = 0.0285$.

Comparing to the sampling period h , we can observe that the average inter-event period h_{avg} has the advantages of reducing the amount of communication and computation. From (4) and (6), this indicates that for both uniform and non-uniform circular formation, the multiple robots under the control law (13) has the properties of order preservation and collision avoidance.

5 Conclusion

In the presence of limited resource availability, this paper investigates a distributed self-triggered circular formation problem for MRS in the plane, where the relationship between robots is modeled as communication topology containing a spanning tree. To be specific, whether the norm of a function with a state exceeds zero, communication between the pairs of neighboring and computation is determined by certain events. It is proved that under the self-triggered mechanism, all robots are driven to form a circular formation surrounding a fixed target and rotate around the target while keeping the desired distance from its neighbors. Moreover, Zeno-behavior is completely excluded. At last, numerical simulation results are given to verify the theoretical analysis. Future work will extend to more complex system structures, such as the effect of space-time topology, weak links in communication networks, and more high-order systems.

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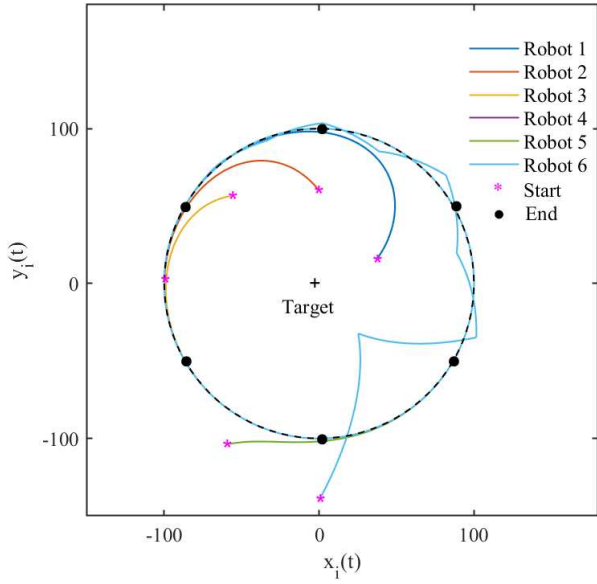


Fig. 3: The trajectories of six robots in the plane at $t \in [0, 200]$.

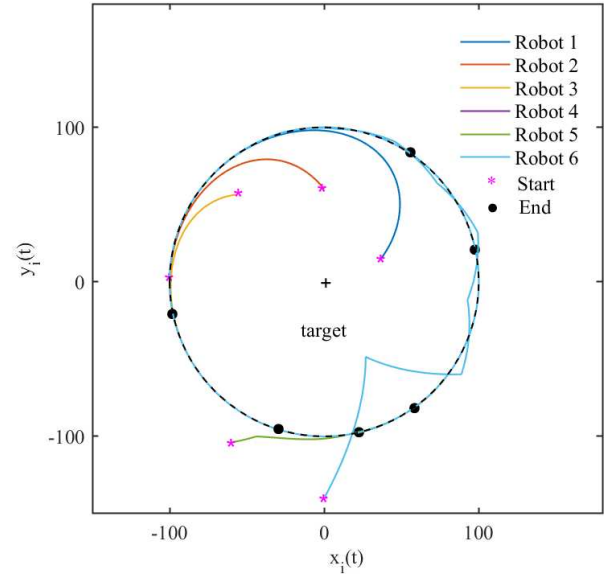


Fig. 5: The trajectories of six robots in the plane at $t \in [0, 200]$.

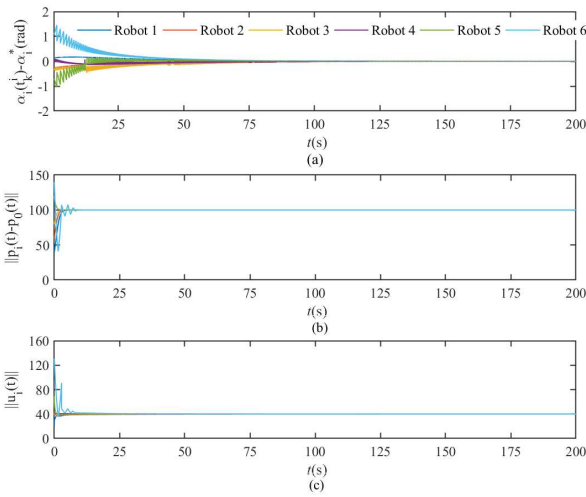


Fig. 4: The evolution of $\|\alpha_i(t_k^i) - \alpha_i^*\|$, $\|p_i(t_k^i) - p_0\|$, $\|u_i(t)\|$ for $i = 1, 2, \dots, 6$.

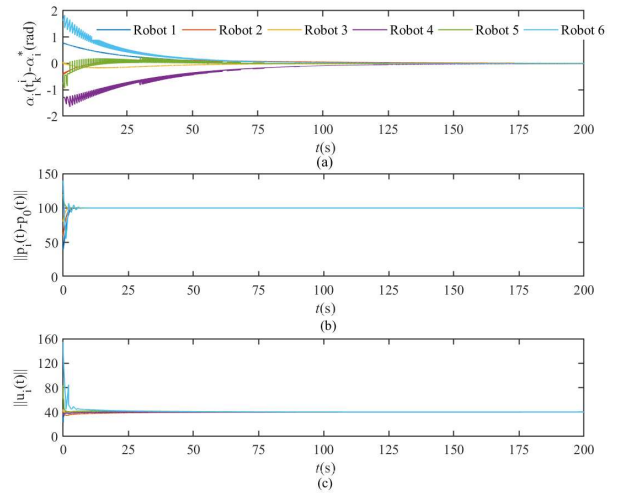


Fig. 6: The evolution of $\|\alpha_i(t_k^i) - \alpha_i^*\|$, $\|p_i(t_k^i) - p_0\|$, $\|u_i(t)\|$ for $i = 1, 2, \dots, 6$.

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